Effect of the tensor force in the exchange channel on the spin-orbit splitting in ^{23}F in the Hartree-Fock framework

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Abstract

We study the spin-orbit splitting (*ls*-splitting) for the proton *d*-orbits in ^{23}F in the Hartree-Fock framework with the tensor force in the exchange channel. ^{23}F has one more proton around the neutron-rich nucleus ^{22}O. A recent experiment indicates that the *ls*-splitting for the proton *d*-orbits in ^{23}F is reduced from that in ^{17}F. Our calculation shows that the *ls*-splitting in ^{23}F becomes smaller by about a few MeV due to the tensor force. This effect comes from the interaction between the valence proton and the occupied neutrons in the $0d_{5/2}$ orbit through the tensor force and makes the *ls*-splitting in ^{23}F close to the experimental data.

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I. INTRODUCTION

The spin-orbit splitting (ls-splitting) is important for the structure of nuclei. A large ls-splitting between single-particle orbits with the same orbital angular momentum is responsible for the shell structure of nuclei [1]. Recently we have been obtaining much information about unstable nuclei from various experiments. There are experimental evidences, which indicate that the shell structure in neutron-rich nuclei changes from that in stable nuclei. To confirm the change of the shell structure, the information about single-particle orbits around closed-shell or sub-closed-shell nuclei is important. Michimasa and his collaborators studied the proton single-particle orbits in ^{23}F experimentally through the proton transfer reaction [2]. ^{23}F has one more proton around ^{22}O. They reported that the ls-splitting for the proton d-orbits ($5/2^+$-$3/2^+$) is 4.06MeV, while the ls-splitting for the proton d-orbits in ^{17}F is 5.00MeV [3, 4], which is similar to the neutron d-orbits in ^{17}O (5.08MeV) [3, 4] due to the isospin symmetry. It indicates that there is a possibility that the ls-splitting is changed by the excess neutrons around ^{16}O. The shell model calculations reproduce the change of the ls-splitting from ^{17}F to ^{23}F nicely [2]. In the shell model calculation the ls-splitting in ^{17}F (^{17}O) is an input parameter. Hence, it is interesting to study the ls-splitting with a mean-field-type model, where the ls-splitting is obtained self-consistently.

Hartree-Fock and Hartree-Fock-Bogoliubov calculations can now be performed in the whole mass region over the nuclear chart. Such mean-field calculations can reproduce binding energies and radii of nuclei including unstable ones using effective forces with relatively simple forms like the Skyrme or Gogny force [5, 6]. In the mean field calculations the ls-splitting of single-particle orbits is produced mainly by the LS force. The ls-splitting of single-particle orbits and the magic number for binding energies can be explained with the LS force having the same strength in almost the whole mass region at least near the stability line. Some studies show that the ls-splitting in neutron-rich nuclei becomes small because the diffuseness of the neutron density becomes large and the spin-orbit potential is weakened [7, 8].

The tensor force acts on the spin of nucleon directly and should affect the ls-splitting. Although the tensor force is not usually included in the mean field calculations, some Hartree-Fock calculations explicitly including the tensor force or the pion in the relativistic model showed that the tensor force affects the ls-splitting in spin-unsaturated nuclei [9, 10, 11, 12,

13, 14, 15, 16]. Only one orbit of the spin-orbit partners is occupied in a spin-unsaturated nucleus, while both the spin-orbit partners are fully occupied in a spin-saturated nucleus. For example, ^{48}Ca is a spin-unsaturated nucleus, where the neutron $0f_{7/2}$ orbit is a spin-unsaturated orbit and ^{40}Ca is a spin-saturated nucleus. Because the total spin coming from the intrinsic spin of nucleon is zero in a spin-saturated nucleus if the wave functions of spin-orbit partners have the same radial forms, the tensor force does not act between the spin-saturated core and a particle or a hole around the core. In a spin-unsaturated nucleus, the total intrinsic spin coming from the spin-unsaturated orbit has a finite value and the tensor force becomes active. In fact, the sizes of the ls-splitting for hole orbits change from ^{40}Ca to ^{48}Ca and ^{16}O to ^{22}O in the results of the Hartree-Fock calculations with the tensor force or the pion [10, 11, 12, 13, 14]. For the calcium isotopes, there is an experimental evidence [17] that the ls-splitting becomes smaller from ^{40}Ca to ^{48}Ca and the order of the change is comparable to that induced by the tensor force or the pion [12, 14]. It should be noted that in the Hartee-Fock approximation the energy contribution from the tensor force or the pion from the direct channel becomes zero and only that from the exchange channel has a finite value in closed-shell nuclei.

Otsuka and his collaborators discussed the effect of the tensor force on single-particle energy in other mass regions. They nicely reproduced the change of the splitting between $\pi 0h_{11/2}$ and $\pi 0g_{7/2}$ in the Sb isotopes with neutron number [18] by the monopole shift induced by the tensor force [14]. They also suggested the effect of the tensor force on the shell evolution in the neutron-rich sd- and pf-shell region [14, 15]. They discussed that the neutron shell structure changes with proton number due to the monopole interaction between proton and neutron orbits and explained the appearance of the magic number 16 and the disappearance of the magic number 20 in the neutron-rich sd-shell region [14, 19, 20]. They claimed that the monopole interaction is caused by the tensor force [14, 15]. To confirm such a discussion, the direct information about a single-particle state is essential.

In this paper we perform the Hartree-Fock calculation for ^{22}O and ^{23}F. We include the tensor force and study its effect on the ls-splitting. We also calculate ^{15,16,17}O to see the effect of valence neutrons on the ls-splitting and its relation to the tensor force by comparing with ^{22}O and ^{23}F. The formulation is given in Section II and the results are given in Section III. Section IV is devoted to the summary of the paper.

II. FORMULATION

In the preset paper we adopt two types of Hamiltonian. One includes the 3-body force in addition to the kinetic term and the two-body force. The other includes the density-dependent force instead of the 3-body force. The Hamiltonian with the 3-body force $H^{3\mathrm{B}}$ and that with the density dependent force H^{DD} have the following forms,

$$H^{3\mathrm{B}} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j}^{A} v(r_i, r_j) + \sum_{i<j<k}^{A} v^{(3)}(r_i, r_j, r_k) - E_{\mathrm{CM}}, \tag{1}$$

$$H^{\mathrm{DD}} = \sum_{i=1}^{A} \frac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j}^{A} v(r_i, r_j) + \sum_{i<j}^{A} v^{(\mathrm{DD})}(\rho; r_i, r_j) - E_{\mathrm{CM}}. \tag{2}$$

In the above expression, $\boldsymbol{p}$, r, and M are the momentum, coordinate including spin and isospin, and mass of nucleon respectively. A is a mass number. v and $v^{(3)}$ are the 2-body and 3-body potentials respectively. $v^{(\mathrm{DD})}$ is the density-dependent potential with the one-body density ρ. We subtract the energy of the center of mass motion $E_{\mathrm{CM}} = (\sum_{i}^{A} \boldsymbol{p}_i)^2 / 2AM$.

In the Hartree-Fock calculation we assume the wave function of the nucleus has the following form,

$$\Psi = \mathcal{A} \prod_{\alpha} \psi_\alpha(r_\alpha) \tag{3}$$

with the antisymmetrization operator $\mathcal{A}$ for nucleon coordinates. α labels each single-particle state and runs over all occupied state. With the wave function the total energies become

$$E^{3\mathrm{B}} = \sum_{\alpha} \langle \psi_\alpha | \frac{\boldsymbol{p}^2}{2M} | \psi_\alpha \rangle + \sum_{\alpha<\beta} \langle \psi_\alpha \psi_\beta | v | \widetilde{\psi_\alpha \psi_\beta} \rangle + \sum_{\alpha<\beta<\gamma} \langle \psi_\alpha \psi_\beta \psi_\gamma | v^{(3)} | \widetilde{\psi_\alpha \psi_\beta \psi_\gamma} \rangle \tag{4}$$

for $H^{3\mathrm{B}}$ and

$$E^{\mathrm{DD}} = \sum_{\alpha} \langle \psi_\alpha | \frac{\boldsymbol{p}^2}{2M} | \psi_\alpha \rangle + \sum_{\alpha<\beta} \langle \psi_\alpha \psi_\beta | v | \widetilde{\psi_\alpha \psi_\beta} \rangle + \sum_{\alpha<\beta} \langle \psi_\alpha \psi_\beta | v^{(DD)}(\rho) | \widetilde{\psi_\alpha \psi_\beta} \rangle \tag{5}$$

for H^{DD}, where the tildes represent the antisymmetrization. In the above equations, E_{CM} is dropped for simplicity. By taking a variation of the total energy with respect to a single-particle wave function ψ_α, we obtain the Hartree-Fock equation for each case:

$$\frac{\boldsymbol{p}^2}{2M} \psi_\alpha(x) + \sum_\beta \int dy \psi_\beta^\dagger(y) v(x, y) \left[\psi_\beta(y) \psi_\alpha(x) - \psi_\alpha(y) \psi_\beta(x) \right]$$

$$+ \frac{1}{2} \sum_{\beta,\gamma} \int dy \int dz \psi_\beta^\dagger(y) \psi_\gamma^\dagger(z) v^{(3)}(x, y, z) \Big[\{ \psi_\beta(y) \psi_\gamma(z) - \psi_\gamma(y) \psi_\beta(z) \} \psi_\alpha(x)$$

$$+ \{ \psi_\gamma(y) \psi_\alpha(z) - \psi_\alpha(y) \psi_\gamma(z) \} \psi_\beta(x) + \{ \psi_\alpha(y) \psi_\beta(z) - \psi_\beta(y) \psi_\alpha(z) \} \psi_\gamma(x) \Big] = \varepsilon_\alpha \psi_\alpha(x) \tag{6}$$

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for the three-body force case and

$$\frac{\boldsymbol{p}^2}{2M}\psi_\alpha(x) + \sum_\beta \int dy\psi_\beta^\dagger(y)v(x,y)\left[\psi_\beta(y)\psi_\alpha(x) - \psi_\alpha(y)\psi_\beta(x)\right]$$

$$+ \sum_\beta \int dy\psi_\beta^\dagger(y)v^{(\mathrm{DD})}(\rho;x,y)\left[\psi_\beta(y)\psi_\alpha(x) - \psi_\alpha(y)\psi_\beta(x)\right]$$

$$+ \sum_{\beta<\gamma} \int dy \int dz\psi_\beta^\dagger(y)\psi_\gamma^\dagger(z)\frac{\delta v^{(\mathrm{DD})}}{\delta\rho}(\rho;y,z)\frac{\delta\rho}{\delta\psi_\alpha^\dagger}(x)\left[\psi_\beta(y)\psi_\gamma(z) - \psi_\gamma(y)\psi_\beta(z)\right] = \varepsilon_\alpha\psi_\alpha(x)$$

$$(7)$$

for the density-dependent force case. In the above expression the integrations over y and z include the summation over the spin and isospin index.

In the present study we assume each single-particle state as an eigenfunction of total spin $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}$. With the assumption a single-particle wave function can be expressed as

$$\psi_\alpha(\boldsymbol{r}) = R_\alpha(r)\mathcal{Y}_{l_\alpha j_\alpha m_\alpha}(\Omega)\zeta(\mu_\alpha), \tag{8}$$

where R is a radial wave function, $\mathcal{Y}$ is an eigenfunction of $\boldsymbol{j}$, and ζ is an isospin wave function. α stands for node n_α, total spin j_α, its projection on the z axis m_α, and isospin μ_α. We do not assume the degeneracy for the orbits with the same n_α, j_α, and μ_α because the spherical symmetry of a mean field is broken in odd nuclei. It means that the states with the same n, j, and μ but different m's are allowed to have different radial wave functions. In such a case we need to perform an angular momentum projection to obtain a wave function with a good angular momentum. The expectation value for the total angular momentum J^2 with the wave function obtained in the Hartree-Fock calculation for a one-particle or one-hole state does not deviate from $j_\nu(j_\nu + 1)$ largely (less than 1%), where j_ν is the total spin of the particle or hole orbit. It indicates the obtained wave function is almost an eigenstate of angular momentum. Hence, we do not perform the angular momentum projection.

We approximate the density in a density-dependent force as

$$\rho(\boldsymbol{r}) \approx \frac{1}{4\pi}\sum_\alpha R_\alpha^\dagger(r)R_\alpha(r) \tag{9}$$

for calculational convenience. This expression is exact for a closed-shell nucleus with the spherical symmetry and should be a good approximation for a one-particle or one-hole nucleus with almost a spherical core.

We expand a radial wave function $R_\alpha(r)$ by Gaussian functions with widths of a geometric series [21]. We take 11 Gaussian functions with the minimum width 0.5fm and the maximum

width 7fm for each single-particle state. The Hartree-Fock equation is solved by the gradient or damped-gradient method [22].

III. RESULT

In this section we apply the Hartree-Fock method to ^{15,16,17,22}O and ^{23}F. We assume ^{16}O as a closed-shell nucleus up to the $0p$-shell and ^{22}O as a sub-closed-shell nucleus where the neutron $0d_{5/2}$ orbit is fully occupied in addition to the occupied orbits in ^{16}O. For ^{22}O there is the experimental evidence which suggests it has the sub-closed-shell structure of the neutron $0d_{5/2}$ orbit [23]. In the ^{15}O case, one neutron is subtracted from the neutron $0p_{1/2}$ orbit or the neutron $0p_{3/2}$ orbit in ^{16}O. In the ^{17}O case we add one neutron in the $0d_{5/2}$ orbit around ^{16}O. We do not put a neutron in the $0d_{3/2}$ orbit in ^{17}O because there are no bound states in this configuration. In the ^{23}F case we add a proton in the $0d_{5/2}$ or $0d_{3/2}$ orbit around ^{22}O.

As for the effective interaction, we adopt the modified Volkov force No. 1 (MV1)[24] for the central part and the G3RS force [25] for the tensor part. We also include the Coulomb force. The G3RS force is determined to reproduce the nucleon-nucleon scattering data and, therefore, the tensor force in the G3RS force is the one in the free space. For the strength of the tensor force in the nuclear medium we do not have a definite guideline at present. The effective interaction obtained from the G-matrix theory has a tensor part with a strength comparable to the tensor force in the free space [14, 26, 27, 28] at least in the region where the relative distance is greater than about 0.8fm. We use the tensor force in the free space in the present calculation but we need a further investigation to determine the strength of the tensor force to be used in a mean field calculation. It should be noted that the difference in the short range ($r < 0.8$fm) does not influence the tensor force matrix elements significantly [28]. As for the LS force we take the δ-type LS force [5, 6]:

$$iW_0(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \overleftarrow{\boldsymbol{k}} \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\overrightarrow{\boldsymbol{k}}. \tag{10}$$

The Majorana parameter in the MV1 force is fixed to 0.59, which is determined to reproduce the binding energy of ^{16}O. W_0 in the LS force is taken as 115MeVfm^5, which is the same as in the Gogny D1 force and is determined to reproduce the ls-splitting for the $0p$ orbits in ^{15}O [6].

In Table I, the results for ^{16}O, ^{17}O, and ^{15}O are summarized. The experimental data

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TABLE I: Total energy (E_{TOT}), kinetic energy (T) and potential energy (V) of ^{16}O, ^{17}O, and ^{15}O. V_{LS} and V_{T} are the contributions from the LS and tensor forces to the potential energy. Those are give in MeV. R_{c} and R_{m} are the charge and matter radii in fm. The last row shows the differences of energies between ^{15}O ($0p^{-1}_{3/2}$) and ^{15}O ($0p^{-1}_{1/2}$). In the parentheses the experimental data are given.

	E_{TOT}	T	V	V_{LS}	V_{T}	R_{c}	R_{m}
^{16}O	-128.3 (-127.6^a)	233.8	-362.0	-1.0	0.0	2.71 $(2.730(25)^b)$	2.58 $(2.54(02)^c)$
^{17}O ($0d_{5/2}$)	-132.3 (-131.8^a)	254.7	-387.0	-4.1	0.0	2.72 $(2.662(26)^b)$	2.64 $(2.59(05)^c)$
^{15}O ($0p^{-1}_{1/2}$)	-110.2 (-112.0^a)	219.6	-329.7	-4.9	-0.1	2.70	2.55 $(2.44(04)^c)$
^{15}O ($0p^{-1}_{3/2}$)	-104.5	212.4	-316.9	0.9	0.0	2.74	2.59
$\Delta(0p^{-1}_{3/2} - 0p^{-1}_{1/2})$	5.7 $(6.18^{\ d})$	-7.2	12.8	5.8	0.1		

^{a}Reference [29].
^{b}Reference [30].
^{c}Reference [31].
^{d}Reference [3, 32].

are also given in the parentheses if available. The potential energy from the tensor force becomes quite small because ^{16}O is a LS-closed-shell nucleus. In the LS-closed-shell nucleus both the spin-orbit partners are completely occupied. Hence, the LS-closed-shell nucleus is a spin-saturated nucleus. The LS-closed-shell nucleus does not have a finite total orbital angular momentum and a finite total spin angular momentum. The tensor force consists of the rank 2 tensors of the orbital and spin angular momenta. Thus, the tensor force does not work between the LS-closed-shell nucleus and a particle or a hole around it, because a particle or hole has a spin angular momentum $1/2$. In the last row the energy differences between ^{15}O ($0p^{-1}_{3/2}$) and ^{15}O ($0p^{-1}_{1/2}$) are shown. It corresponds to the ls-splitting for the $0p$ orbits. It is about 10% smaller than the experimental value. The contribution from the LS force is 5.8MeV and is almost the same as the total ls-splitting. It indicates that the ls-splitting is mainly produced by the LS force. The large contribution from the kinetic energy is almost canceled out with the contributions from the central and three-body forces. In ^{15}O the effect of the tensor force on the ls-splitting is negligible.

In Table II the results for ^{22}O and ^{23}F are summarized. Although the binding energy

TABLE II: Total energy (E_{TOT}), kinetic energy (T) and potential energy (V) of ^{22}O and ^{23}F. V_{LS} and V_{T} are the contributions from the LS and tensor forces to the potential energy. Those are give in MeV. R_{c} and R_{m} are the charge and matter radii in fm. The last row is the differences of energies between ^{23}F ($0d_{3/2}$) and ^{23}F ($0d_{5/2}$). In the parentheses the experimental data are given.

	E_{TOT}		T	V	V_{LS}	V_{T}	R_{c}	R_{m}	
^{22}O	-161.8	(-162.0^a)	361.4	-523.2	-20.8	1.9	2.74	2.85	$(2.88(06)^b)$
^{23}F ($0d_{3/2}$)	-166.5		376.4	-542.8	-16.3	0.1	2.89	2.90	
^{23}F ($0d_{5/2}$)	-170.7	(-175.3^a)	383.9	-554.5	-24.1	3.2	2.84	2.87	$(2.79(04)^b)$
$\Delta(0d_{3/2} - 0d_{5/2})$	4.2	(4.06^c)	-7.5	11.7	7.8	-3.1			

^{a}Reference [29].
^{b}Reference [31].
^{c}Reference [2].

of ^{23}F ($0d_{5/2}$) (the ground state) is about 5MeV smaller than the experimental value, it probably does not affect our discussion on the ls-splitting. In ^{22}O the neutron $0d_{5/2}$ orbit around the ^{16}O core is fully occupied. Because the spin-orbit partner, the neutron $0d_{3/2}$ orbit, is empty, ^{22}O is a spin-unsaturated nucleus. Hence, ^{22}O has a finite total orbital angular momentum and a finite total spin angular momentum, and the expectation value for the tensor potential energy in ^{23}F becomes finite. In ^{22}O, the energy contributions from the LS force and the tensor force are -20.8MeV and 1.9MeV respectively. In ^{23}F a proton is added to ^{22}O. If the proton is put in the $0d_{3/2}$ orbit the absolute value of the LS potential energy becomes small by 4.5MeV and if the proton is put in the $0d_{5/2}$ orbit that of the LS potential energy becomes large by 3.3MeV. In contrast, the tensor potential energy becomes small by 1.8MeV when the proton is in the $0d_{3/2}$ orbit and becomes large by 1.3MeV when the proton is in the $0d_{5/2}$ orbit. As a result, the contribution to the ls-splitting for the proton $0d$ orbits in ^{23}F from the LS force is 7.8MeV and that from the tensor force is -3.1MeV. The sum of them is 4.5MeV. The relatively small ls-splitting 4.2MeV after adding the contributions from the kinetic and other potential energies, which is close to the experimental value, is realized by the cancelation between the contributions from the LS force and the tensor force.

The energy differences between one-particle states and their corresponding cores are shown in Table III. The LS potential energies from the cores for the $0d_{5/2}$ orbit are -3.0MeV

TABLE III: Differences of the LS potential energy ($\Delta(V_{\mathrm{LS}})$) and the tensor potential energy ($\Delta(V_{\mathrm{T}})$) between one-particle nuclei and their core nuclei. Those are given in MeV.

	$\Delta(V_{\mathrm{LS}})$	$\Delta(V_{\mathrm{T}})$
^{17}O$(0d_{5/2})-^{16}$O	-3.0	0.0
^{23}F$(0d_{3/2})-^{22}$O	4.5	-1.8
^{23}F$(0d_{5/2})-^{22}$O	-3.3	1.3

in ^{17}O and -3.3MeV in ^{23}F. The LS potential energy for the $0d_{3/2}$ orbit in ^{23}F is smaller as expected from that for the $0d_{5/2}$ orbit ($3.3 \times (2+1)/2 \approx 5.0$MeV). It is probably due to a weak binding of the $0d_{3/2}$ orbit compared to the $0d_{5/2}$ one. The contribution from the tensor force to the splitting for the $0d$ orbits in ^{23}F is about a half of that from the LS force with the opposite sign as discussed in the previous section. The results for ^{17}O and ^{23}F in Table III indicate that the contribution to the ls-splitting from the LS force mainly comes from the ^{16}O core and that from the tensor force comes from the excess neutron orbit (the neutron $0d_{5/2}$ orbit).

TABLE IV: Potential energy contributions from the triplet-even tensor force ($V_{\mathrm{T}}^{3\mathrm{E}}$) and the triplet-odd tensor force ($V_{\mathrm{T}}^{3\mathrm{O}}$) in MeV. In the last two rows, the differences between ^{23}F ($0d_{3/2}$ or $0d_{5/2}$) and ^{22}O are given.

	$V_{\mathrm{T}}^{3\mathrm{E}}$	$V_{\mathrm{T}}^{3\mathrm{O}}$
^{22}O	0.1	1.8
^{23}F $(0d_{3/2})$	-1.3	1.4
^{23}F $(0d_{5/2})$	1.0	2.1
$\Delta(^{23}$F$(0d_{3/2})-^{22}$O$)$	-1.4	-0.4
$\Delta(^{23}$F$(0d_{5/2})-^{22}$O$)$	1.0	0.3

In Table IV, the contributions to the tensor potential energy from the triplet-even and triplet-odd parts are shown separately. In ^{22}O the tensor potential energy mainly comes from the triplet-odd part. It is natural because only the neutron $0d_{5/2}$ orbit is occupied and there are no valence protons around the ^{16}O core. In ^{23}F the contribution from the triplet-even part is comparable to that from the triplet-odd part for the $0d_{3/2}$ orbit and they

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have the opposite sign. For the $0d_{5/2}$ orbit the contribution from the triplet-even part is smaller than that from the triplet-odd part and they have the same sign. To see the effect of the tensor force on the valence proton, the energy differences between ^{23}F and ^{22}O are shown in the table. The differences are dominated by the triplet-even part. It means that the contribution to the ls-splitting from the tensor force mainly comes from the triplet-even tensor force.

TABLE V: ls-splitting for the proton d-orbits in ^{23}F with various effective interactions (see the text). $\Delta(V_{\mathrm{LS}})$, $\Delta(V_{\mathrm{T}})$, and $\Delta(\text{others})$ are the contributions to the ls-splitting from the LS force, the tensor force, and the other forces including the kinetic term respectively. Those are given in MeV. The experimental value for $\Delta(0d_{3/2} - 0d_{5/2}) = 4.06$MeV.

	$\Delta(0d_{3/2} - 0d_{5/2})$	$\Delta(V_{\mathrm{LS}})$	$\Delta(V_{\mathrm{T}})$	$\Delta(\text{others})$
MV1	4.2	7.8	−3.1	−0.5
MV1 without V_{T}	7.2	8.3	0.0	−1.1
Gogny D1S	8.5	9.4	0.0	−0.9
M3Y-P2	7.6	9.2	−0.4	−1.2
GT2	8.2	12.2	−3.3	−0.7

Finally we compare the ls-splitting calculated with other effective interactions with our result discussed above (MV1) in Table V. We also show the result without the tensor force (MV1 without V_{T}). The Gogny D1S force [33] does not have a tensor part and a stronger LS part (W_0=130MeVfm^5) than one we adopted above. The M3Y-P2 force [34] has a weak tensor part and an LS part comparable to the Gogny D1S force. The GT2 [15, 35] force has a tensor part comparable to that in the free space and a strong LS part (W_0=160MeVfm^5). While the rather schematic form of the tensor force is adopted in Ref. 15, we replace the tensor part of the GT2 force with the G3RS force we used above. The sizes of the ls-splitting for the MV1 force without the tensor force, the Gogny force, and the M3Y-P2 force are large compared to the experimental value. It indicates that the relatively strong tensor force comparable to that in the free space is needed to reproduce the ls-splitting in ^{23}F. Although the GT2 force has a strong tensor part, it gives quite large splitting. It is due to the strong LS part of the GT2 force. The contribution from the LS force to the ls-splitting is much larger than those with other effective forces. In fact, the ls-splitting for the $0p$

orbits in ^{15}O with the GT2 force is 8.3MeV. It is much larger than the experimental value. It indicates that the proper strength of the LS force, which give the reasonable ls-splitting in ^{15}O is needed to reproduce the ls-splitting in ^{23}F.

The tensor force also induces a 2-particle–2-hole ($2p2h$) correlation, which cannot be treated in a usual mean field calculation. The $2p2h$ correlation by the tensor force produces the large attractive energy in nuclei [36, 37]. Recently we developed a mean field framework which can treat the $2p2h$ tensor correlation by introducing single-particle states with charge and parity mixing [38, 39, 40, 41]. We applied the extended mean field model to sub-closed-shell oxygen isotopes [41] and found that the potential energy from the tensor force is comparable to that from the LS force. The importance of the $2p2h$ tensor correlation for the ls-splitting is indicated in other studies [40, 42, 43, 44]. It is interesting to study the effect of the $2p2h$ tensor correlation on the ls-splitting with our extended mean filed model. Because our calculation showed that the excess neutrons around ^{16}O do not contribute to the $2p2h$ tensor correlation strongly [41], the Hartree-Fock calculation seems to be sufficient as the first step.

IV. SUMMARY

We have performed the Hartree-Fock calculation with the tensor force for ^{15}O, ^{16}O, ^{17}O, ^{22}O, and ^{23}F to study the effect of the tensor force on the ls-splitting.

The tensor force does not affect the ls-splitting for the $0p$ orbits in ^{15}O because ^{16}O is a LS-closed-shell nucleus. The ls-splitting is almost produced by the LS force in ^{15}O.

In ^{22}O, the neutron $0d_{5/2}$ orbit is fully occupied. It gives the finite expectation value for the tensor force in ^{22}O. In ^{23}F a proton is added to ^{22}O. The LS force works to provide the ls-splitting for the proton $0d$-orbits in ^{23}F by 7.8MeV. In contrast, the tensor force reduces the ls-splitting by 3.1MeV. The effect of the tensor force mainly comes from the occupied neutron $0d_{5/2}$ orbit. The resulting ls-splitting of 4.2MeV close to the experimental data is realized by the cancelation between the effects of the LS force and the tensor force. The contribution from the tensor force to the ls-splitting in ^{23}F mainly comes from the triplet-even part of the tensor force.

We have compared the results with various effective interactions with and without the tensor force. The effective interaction without the tensor force or with the weak tensor force

does not explain the experimental value for the ls-splitting for the proton $0d$-orbits in ^{23}F. Our study indicates that the LS and tensor forces with reasonable strengths are needed to reproduce the ls-splitting in ^{15}O and ^{23}F, simultaneously.

Acknowledgments

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